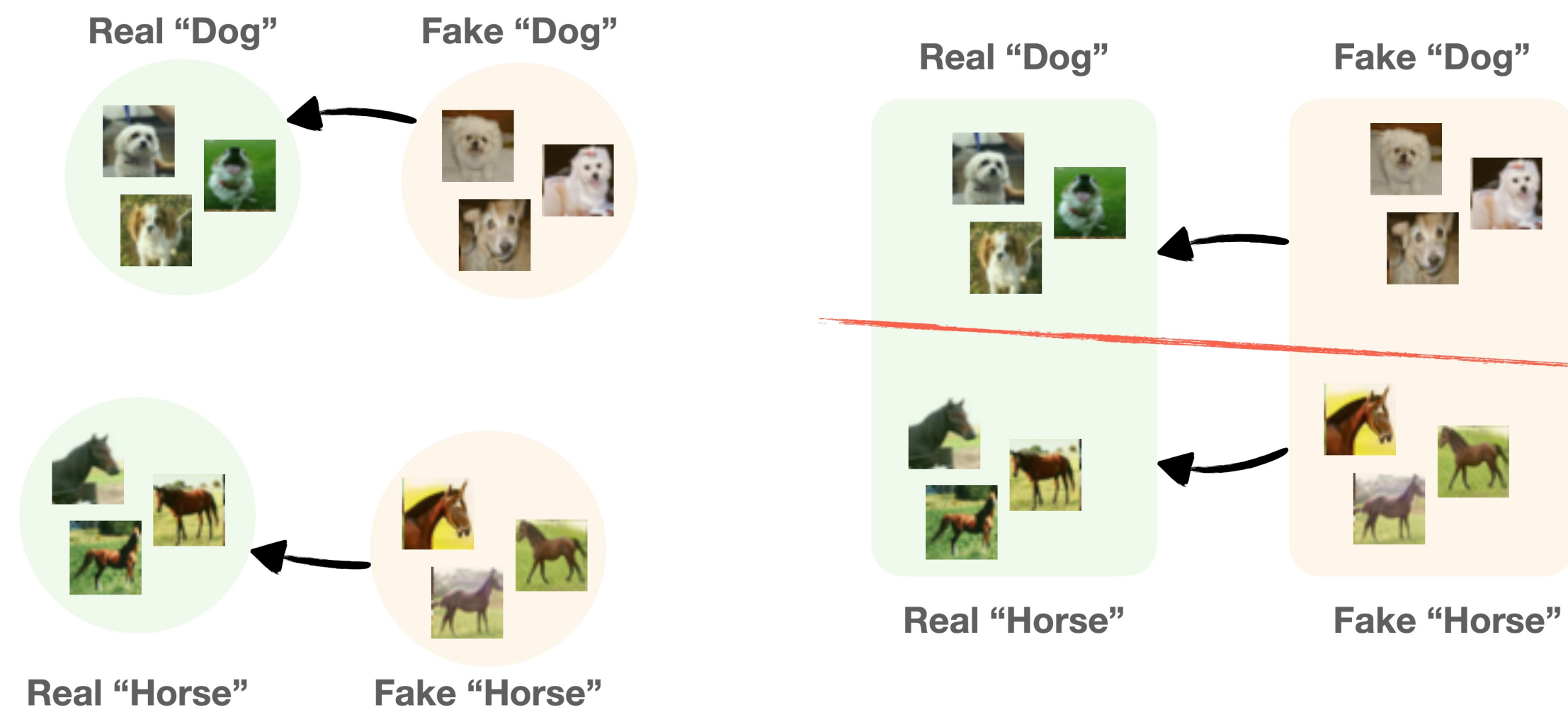


## Introduction

- Data Matching and Label Matching
- Conditional Data Matching

Marginal Matching + Label Matching



- Conditional GAN
- Objectives

$$L_D = \mathbb{E}_{x,y \sim P_{XY}} \mathcal{A}(-\tilde{D}(x,y)) + \mathbb{E}_{z \sim P_Z, y \sim Q_Y} \mathcal{A}(\tilde{D}(G(z,y), y))$$

$$L_G = \mathbb{E}_{z \sim P_Z, y \sim Q_Y} \mathcal{A}(-\tilde{D}(G(z,y), y))$$

$\mathcal{A}$  is activation function,  $\tilde{D}$  is the logit. Vanilla GAN,  $\mathcal{A}(t) = \text{softplus}(t) = \log(1 + \exp(t))$ .

- Decomposition

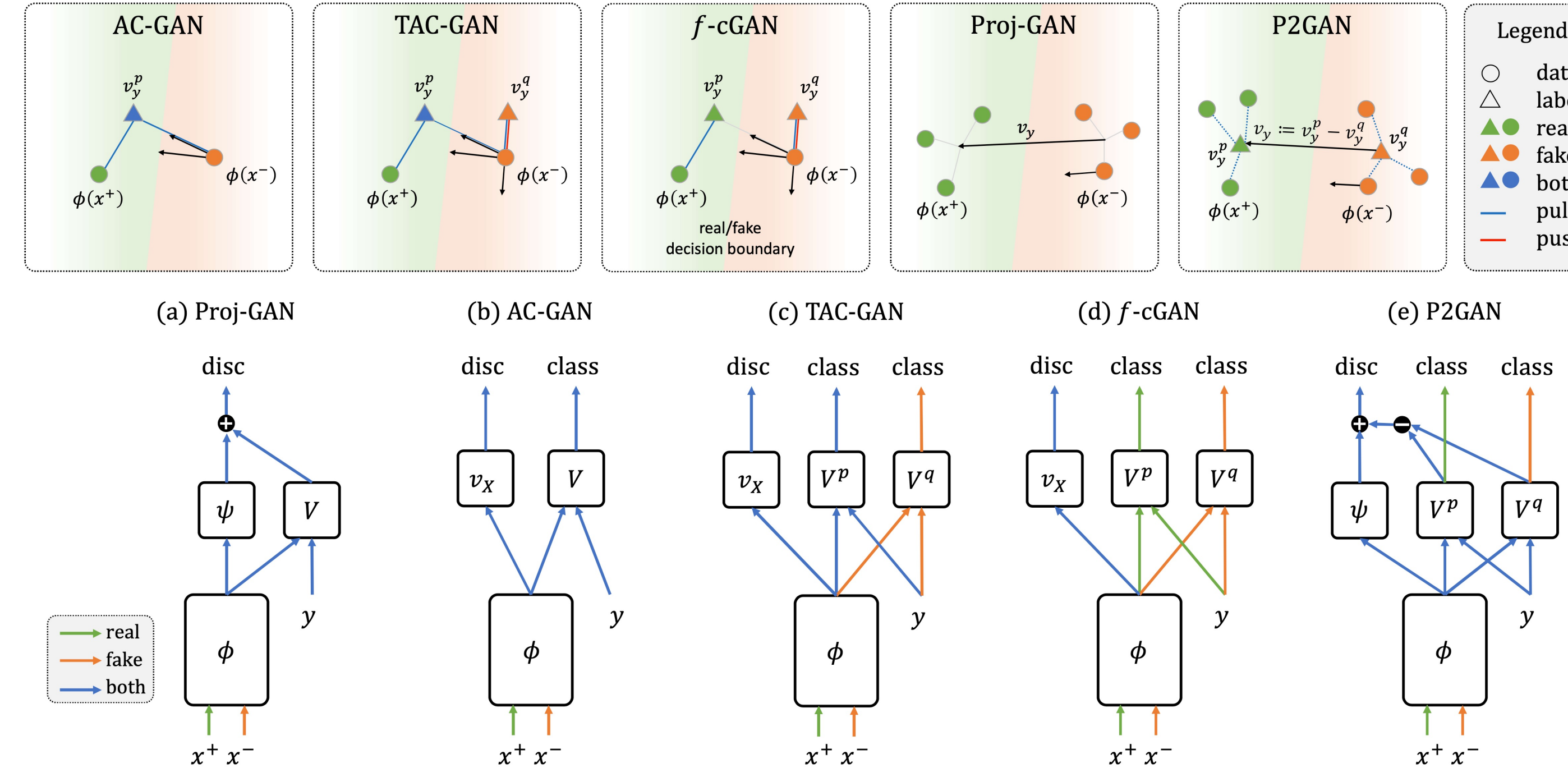
$$\begin{aligned} \tilde{D}^*(x,y) &= \log \frac{P(x)}{Q(x)} + \log \frac{P(y|x)}{Q(y|x)} \\ &= \underbrace{\log \frac{P(x|y)}{Q(x|y)}}_{\text{Marginal Matching}} + \underbrace{\log \frac{P(y)}{Q(y)}}_{\text{Label Matching}} \\ &= \underbrace{\log \frac{P(x|y)}{Q(x|y)}}_{\text{Data Matching}} + \log \frac{P(y)}{Q(y)} \end{aligned}$$

- Projection GAN (Miyato 2018)

$$\tilde{D}(x,y) = v_y^T \phi(x) + \psi(\phi(x))$$

$\phi(\cdot)$  is the image embedding function,  $v_y$  is embedding of class.  $v_y := v_y^p - v_y^q$  is the difference of real and fake class embeddings.

## Dual Projection GAN (P2GAN)



- Objective

$$\tilde{D} = (v_y^p - v_y^q)^T \phi(x) + \psi(\phi(x))$$

Untying Class Embeddings

$$L_D^{P2} = L_D(\tilde{D}) + L_{mi}^p + L_{mi}^q \quad \text{Classification Losses}$$

$$L_G^{P2} = L_G(\tilde{D})$$

- Weighted Dual Projection GAN (P2GAN-w)

$$L_D^{P2w} = L_D + \lambda \cdot (L_{mi}^p + L_{mi}^q)$$

## Theoretical Analysis

**Proposition 1.** When  $\psi = 0$ , a Proj-GAN reduces to  $K$  unconditional GANs, each of them minimizes the Jensen-Shannon divergence between  $P_{X|y}$  and  $Q_{X|y}$  with mixing ratio  $\{\frac{P(y)}{P(y)+Q(y)}, \frac{Q(y)}{P(y)+Q(y)}\}$ . Its value function can be written as,

$$\mathbb{E}_{P_Y} \left\{ \mathbb{E}_{P_{X|Y}} \log D(x|y) + \frac{Q_Y}{P_Y} \mathbb{E}_{Q_{X|Y}} \log(1 - D(x|y)) \right\}$$

**Proposition 2.** Given a generator  $G$ , if cross entropy losses  $L_{mi}^p$  and  $L_{mi}^q$  are minimized optimally, then the difference of two losses evaluated at fake data equals the reverse KL-divergence between  $P_{Y|X}$  and  $Q_{Y|X}$ ,

$$L_{mi}^p(x^-) - L_{mi}^q(x^-) = \mathbb{E}_{Q_X} KL(Q_{Y|X} \| P_{Y|X}). \quad (10)$$

### DM-GAN (explicit data matching)

- Set  $\psi(\cdot) = 0$

### f-cGAN (explicit label matching)

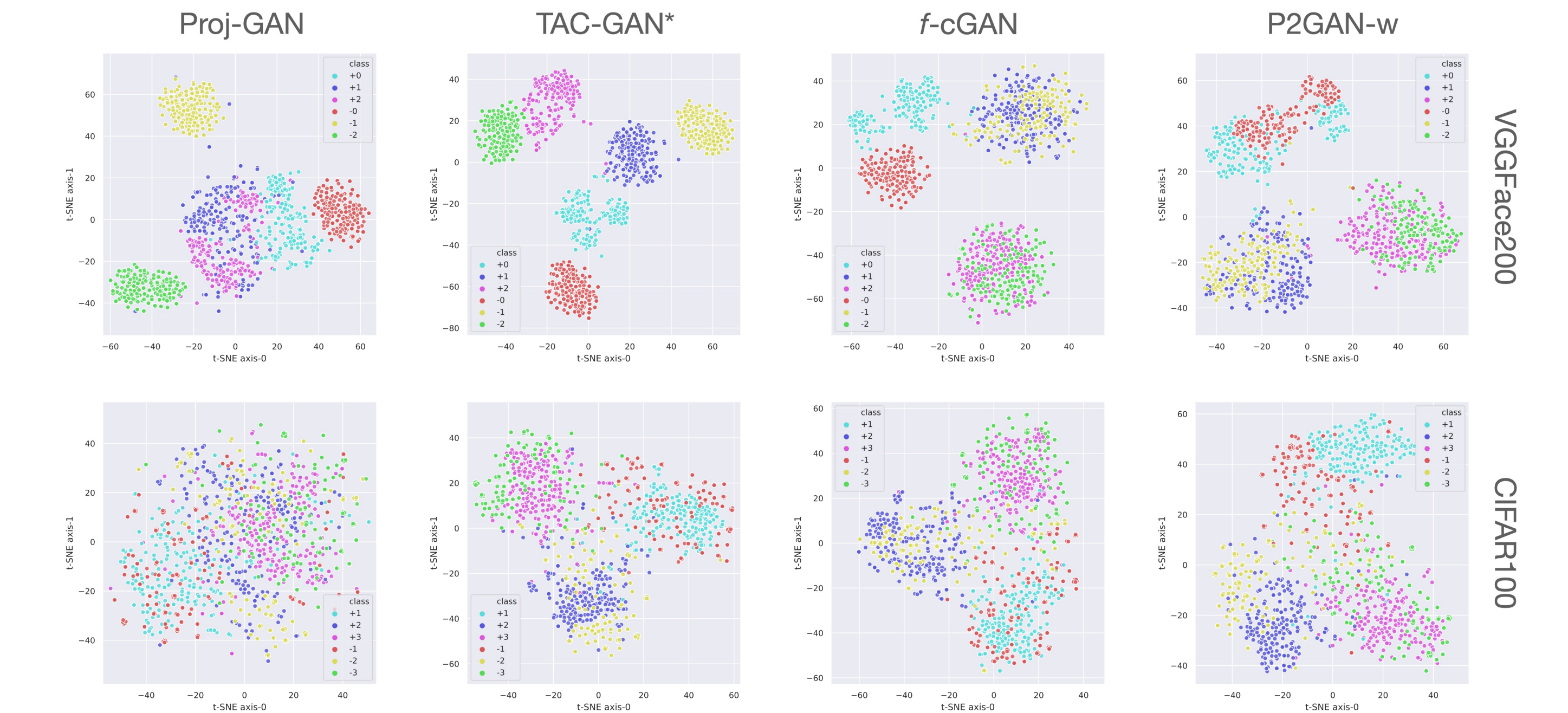
$$L_D^f = L_D(\tilde{D}) + L_{mi}^p(x^+) + L_{mi}^q(x^-)$$

$$L_G^f = L_G(\tilde{D}) + L_{mi}^p(x^-) - L_{mi}^p(x^+)$$

$$\tilde{D} = v_X^T \phi(x) + b$$

## Experiments

- VGGFace2 (200 identity subset)



- ImageNet and CIFAR10

	ImageNet 128 × 128		CIFAR10	
	IS ↑	FID ↓	IS ↑	FID ↓
Proj-GAN	30.73	23.07	*9.85	*8.03
P2GAN	<b>59.24</b>	<b>16.86</b>	9.76	8.00
P2GAN-w	42.69	19.19	<b>9.87</b>	<b>7.99</b>

## Summary

We give insights on projection form of conditional discriminators and propose a new conditional generative adversarial network named Dual Projection GAN (P2GAN). We demonstrate its flexibility in modeling and balancing data matching and label matching. We further rigorously analyze the underlying connections between AC-GAN, TAC-GAN, and Proj-GAN. From the analysis, we first propose f-cGAN, a general framework for learning conditional GAN. We demonstrate the efficacy of our proposed models on various synthetic and real-world datasets.

